Finding the Midpoint and Length of a Line Segment

In coordinate geometry, it is important to be able to find the midpoints and the lengths of segments.

EXAMPLE A

a. Find the coordinates of the midpoints of $TP$ and $AR$.

b. Compare the length of $MN$ with the sum of the lengths of $TR$ and $PA$.

Solution:

a. For the segment whose endpoints are $(x_1, y_1)$ and $(x_2, y_2)$, the coordinates of the midpoint are $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

$M: \left(\frac{-6 + (-6)}{2}, \frac{9 + 1}{2}\right) = \left(\frac{-12}{2}, \frac{10}{2}\right) = (-6, 5)$

$N: \left(\frac{0 + 12}{2}, \frac{-2 + 0}{2}\right) = \left(\frac{12}{2}, \frac{-2}{2}\right) = (6, -1)$

b. Use the Distance Formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to find the distance between $(x_1, y_1)$ and $(x_2, y_2)$.

$MN = \sqrt{(-6 - 6)^2 + (5 - 1)^2} = \sqrt{(-12)^2 + 6^2} = \sqrt{144 + 36} = \sqrt{180} = \sqrt{36 \cdot 5} = 6\sqrt{5}$ units

$TR = \sqrt{(-6 - 12)^2 + (9 - 0)^2} = \sqrt{324 + 81} = \sqrt{405} = \sqrt{81 \cdot 5} = 9\sqrt{5}$ units

$PA = \sqrt{(-6 - 0)^2 + (1 - (-2))^2} = \sqrt{36 + 9} = \sqrt{45} = 3\sqrt{5}$ units

Notice that the average of $9\sqrt{5}$ and $3\sqrt{5}$ is $\frac{9\sqrt{5} + 3\sqrt{5}}{2} = \frac{12\sqrt{5}}{2} = 6\sqrt{5}$. In other words, the length of $MN$, which is $6\sqrt{5}$ units, is the average of the lengths of $TR$ and $PA$; or the length of $TR$ and $PA$ together, is twice the length of $MN$. 
In $\triangle RST$, points $V$ and $W$ are the midpoints of $RT$ and $ST$. Compare the lengths of $RS$ and $VW$.

**Step 1:** Use $R(1, 7)$ and $S(5, 1)$ to find the length of $RS$.

$$RS = \sqrt{(1 - 5)^2 + (7 - 1)^2} = \sqrt{16 + 36} = \sqrt{52} = 2\sqrt{13}$$

**Step 2:** Use the Midpoint Formula to find coordinates of points $V$ and $W$.

$V: \left(\frac{-3 + 1}{2}, \frac{-3 + 7}{2}\right) = (\frac{-2}{2}, \frac{4}{2}) = (-1, 2)$

$W: \left(\frac{-3 + 5}{2}, \frac{-3 + 1}{2}\right) = (\frac{2}{2}, \frac{-2}{2}) = (1, -1)$

**Step 3:** Use $V(-1, 2)$ and $W(1, -1)$ to find the length of $VW$.

$$VW = \sqrt{(-1 - 1)^2 + (2 - (-1))^2} = \sqrt{4 + 9} = \sqrt{13}$$

**Solution:** The length of $VW$ is half the length of $RS$. This actually illustrates the Triangle Midsegment Theorem, which states that the segment joining the midpoints of two sides of a triangle is parallel to the other side of the triangle and is half the length of that side.
Finding the Midpoint and Length of a Line Segment (continued)

PRACTICE

Use the diagram shown for Items 1–2.

1. Find the midpoints of $MT$ and $AH$.
2. Find the lengths of $MT$ and $AH$.

Use the diagram shown for Items 3–7.

3. Find the coordinates of the midpoint of $AC$. Do the coordinates satisfy $y = -x + 8$? Is the midpoint of $AC$ on the line $y = -x + 8$?
4. Find the coordinates of the midpoint of $BC$.
5. Find the coordinates of the midpoint of $AB$.

6. What is the length of the segment from the midpoint of $AB$ to vertex C?

7. Find the distance between points C and R. How does that distance compare to the length of the segment from the midpoint of $AB$ to vertex C?